

# Development of the Monte-Carlo method to predict radiative heat transfer within the boilers and furnaces

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## SUMMARY

Comparison of various methods of radiative heat transfer calculations with the Monte Carlo procedure (MCP) is carried out in frame of some principal geometries of the emitting medium. The detailed conditions of MCP performance to conform the results with those in accordance with the zone and flux methods are discussed for infinite flat layer, boiler furnace, industrial and process furnaces. Adequacy of the model of the weighted sum of grey gases (WSGG) is tested to compute the gas medium radiation in combination with proposed MCP version.

## INTRODUCTION

The method of statistical tests or Monte-Carlo procedure has taken a wide spread by furnace equipment calculations and by high-temperature firing units (boilers and furnaces) improvement in frame of solution of following tasks [1,2,3]:

- calculation of local and average angular radiation factors in diathermic medium;
- definition of generalized and resolving angular factors of radiation, including those accounting anisotropic scattering being used at zone calculation methods of radiative heat exchange;
- calculation of the spectral (for grey gas – of integral) emissivity factor of the radiating gas volume.

Calculation of mentioned angular factors corresponds to determination of geometrical factors and the surfaces of a mutual radiative exchange: direct and total – between the computation zones of both surface and volume types within the framework of Hottel's zone method [4].

In generalizing P. Lybaert's report [5] Monte-Carlo method is mentioned among other methods of definition of direct surfaces of a mutual exchange, including analytical (being offered for the simple geometries of the system) and alternative procedures – the discrete transfer methods (of the fixed directions of the beams being suitable practically for any system). Unlike the Monte-Carlo method, the simplified procedures cannot be used for calculation of the generalized angular factors and of emissivity factor of radiating volumes.

## 1. COMPARISON OF CALCULATIONS FOR A FLAT RADIATING LAYER

Our earlier calculations have been submitted and generalized in the monograph book [6] basing on some procedures: of multiple (repeated) reflections, of integro-exponential  $E$ -functions, the approximate method of effective fluxes – are compared below with the new computations providing an examination of adequacy of Monte-Carlo procedure (MCP).

In our presentations [7,8] the Monte-Carlo procedure was developed for the case of general calculation of the combustion chambers and the boiler furnaces filled in of the emitting and absorbing medium. The last was considered in frame of approximation of non-grey radiation being considered as the weighted sum of grey gases (WSGG). This approach is used within present paper.

Selectivity and non-isothermal state of the gas volume of non-grey gases were taken into account by use of H. Hottel's model [7,8] and by representation of the integral emissivity factor  $\varepsilon_g$  and absorption abilities  $A_g$  of real gas (combustion products) by means of weighted sum of radiation of  $n$  grey gases each of them being characterized by absorption factor  $k_n$  not depending on temperature. Contribution of the  $n$ -th gas radiation depends on gas  $T_g$  or emitter  $T_s$  temperature, the corresponding fraction  $a_n$  being submitted to a condition of normalization:

$$\varepsilon_g = \sum_n a_n (T_g) (1 - e^{-k_n p_\Sigma \Delta l_{ef}}); \quad (1)$$

$$A_g = \sum_n a_n (T_s) (1 - e^{-k_n p_\Sigma \Delta l_{ef}}); \quad (2)$$

$$\sum_n a_n = 1, \quad (3)$$

where  $p_\Sigma = p_{CO_2} + p_{H_2O}$ ;  $K_n = k_n p_\Sigma$

Various approximations for non-grey radiation are collected in the literature where the number  $n$  makes value between 3 up to 6 components [5] and even up to 19 [9]. The  $a_n$  and  $k_n$  values for the combustion products of natural gas ( $p_{H_2O} = 19$  kPa,  $p_{CO_2} = 9.5$  kPa) were taken accordingly Taylor and Foster data [10] by number  $n = 4$  in our calculations have been performed earlier and within the framework of the present presentation.

We'll call the  $n$ -th grey gas contribution in case of non-grey gas as  $n$ -th "quasi-level" by analogy with the structure of the selectively emitting gas. Using Monte-Carlo procedure accordingly [11] the different random numbers must correspond to each of "quasi-levels" of non-grey radiation. This approach simulates subjecting of the selective radiation characteristics to stochastic computation procedures.

By our present performance the approach is changed as contrasted to the paper [11]: the complete cycles of statistical trials are fulfilled for each of  $n$  grey gases in the same manner. The similar procedure was recommended in the manual [12] for separate spectral intervals of selectively radiating gas.

The difference of the method, originated by us, from those in the book [12] is, that we have fulfilled the straight calculation of resulting heat fluxes by radiative transfer between all volume and surface zones while in [12] the spectral resolvent angular coefficients (SRAC) were computed basing upon optical and geometrical performance between volume and surface zones inherent to the enclosure. The direct interchange areas represent analogue of SRAC by use of H. Hottel's zone method grounded upon WSGG model of radiating gas.

The computation area represents two infinite parallel plates: the radiator (emitter) – an adiabatic brickwork of unknown temperature  $T_R$  and the receiver of given temperature  $T_A$  - enclosing the flat gas layer of  $\Delta l_\Sigma$  thickness, the last being filled in of the completed combustion products of natural gas. Calculation of the resulting specific heat flux  $q_A$  and refractory temperature  $T_R$  makes the problem to be solved.

Table 1. Temperature pattern of the gas layer sections across the gas flat volume within the furnace by various heat transfer conditions: uniform distribution, direct, indirect, direct + indirect heating (in accordance with classification [6]).

Section site #	Direct + indirect	Direct			Indirect				Uniform distribution	
	Variant									
	1	2	3	4	5	6	7	8	9	
1	1700	1300	1300	1700	1700	1600	1570	1400	1300	
2	1570	1330	1330	1570	1655	1570	1700	1400	1305	
3	1470	1335	1335	1475	1625	1550	1475	1400	1310	
4	1380	1340	1340	1445	1595	1530	1445	1400	1315	
5	1330	1345	1345	1420	1565	1510	1420	1400	1320	
6	1320	1350	1350	1405	1535	1490	1405	1400	1330	
7	1315	1355	1355	1395	1505	1470	1395	1400	1380	
8	1310	1360	1360	1385	1475	1450	1385	1400	1470	
9	1305	1365	1365	1380	1445	1430	1380	1400	1570	
10	1300	1370	1370	1375	1415	1410	1375	1400	1700	
11	1300	1375	1375	1370	1385	1390	1370	1400	1700	
12	1305	1380	1380	1365	1355	1370	1365	1400	1570	
13	1310	1385	1385	1360	1325	1350	1360	1400	1470	
14	1315	1395	1395	1355	1295	1330	1355	1400	1380	
15	1320	1405	1405	1350	1265	1310	1350	1400	1330	
16	1330	1420	1420	1345	1235	1290	1345	1400	1320	
17	1380	1445	1445	1340	1205	1270	1340	1400	1315	
18	1470	1475	1475	1335	1175	1250	1335	1400	1310	
19	1570	1570	1700	1330	1145	1230	1330	1400	1300	
20	1700	1700	1570	1300	1100	1200	1300	1400	1300	

Temperature  $T_g$  profile across the gas layer is submitted in the Table 1. The gas layer is divided upon  $N = 20$  discrete sections of the same thickness. Numbering of the sections ( $i = 1$ ) is beginning from the layer, being the nearest to the refractory wall. The layer contacting with the receiver wall is numbering as  $i = N = 20$ ). Various temperature profiles were compared from the efficiency standpoint (estimation of the maximal  $q_A$  value) [6]. The temperature profiles were assigned in such a manner that an averaged temperature value was conserved for any of nine variants being under consideration:

$$\bar{T}_g = \frac{1}{\Delta l_{\Sigma} (\Delta l_{\Sigma})} \int T_g dl = 1400 K = idem \quad (4)$$

It was supposed, that the resulting heat flux being absorbed by enclosing surfaces  $S \in \{A, R\}$  includes radiative  $q_{rad}^S$  and convective  $q_{conv}^S$  components in frame of additivity ability:  $q_S = q_{rad}^S + q_{conv}^S$ , and  $q_R = 0$ . The surfaces  $A$  and  $R$ , enclosing the gas layer, were assumed as grey bodies by performance of calculation. Other conditions and limitations are similar to those assumed in book [6].

MCP computations were carried out by substitution of infinite flat layer for finite sizes one surrounded by mirror reflected side surfaces. The results of comparison of the resulting heat fluxes  $q_A$  for various temperature profiles across the layer are demonstrated by Fig.1 and are fairly coincided between two computation procedures under consideration.

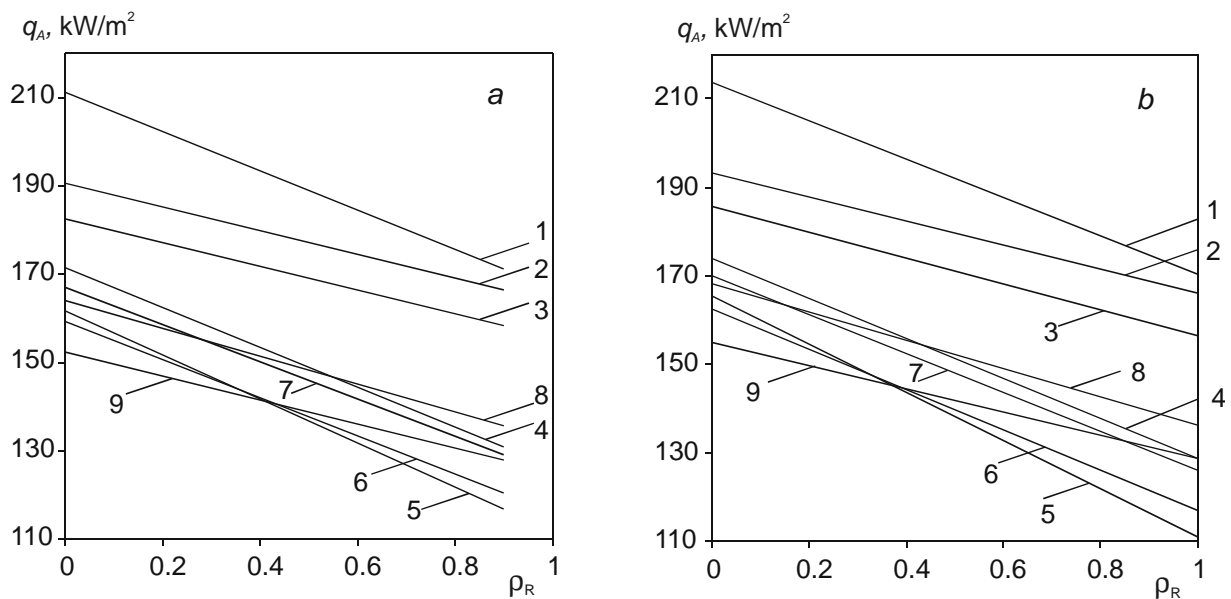


Fig.1. Dependence of resulting heat flux density  $q_A$  absorbed by receiving surface of flat layer of natural gas combustion products in dependence on reflectivity of refractory surface  $\rho_R$ .

The number of the curves corresponds to the variant of temperature pattern across the flat layer of natural gas' combustion products (Table 1)

a – our calculations by Monte-Carlo procedure. Relative energy remainder  $\xi = 0.002$ ;

b – our data being computed by multiple reflections procedure and adopted from [6].

## 2. COMPARISON OF CALCULATIONS FOR THE BOILERS' AND FURNACES' COMBUSTION CHAMBERS

Reliability of performance of the Monte-Carlo procedure with reference to heat exchange calculations within the boilers and furnaces is of great practical interest. The following typical designs of the furnaces have been chosen to testify MCP

- the parallelepipeds have been extended in height, schematising the furnaces of the steam generating units and of the tubular process furnaces (of oil refining and of chemical branches of industry),

- the parallelepipeds have been extended in horizontal planes, schematising the reheating furnaces.

**2.1.** The following geometrical ratios are accepted for the units calculated by H. Kremer:  $l = b$ ,  $h = 2.5b$ , the range of the size  $b$  variation – up to 15 m, the step of change – 5 m – for the boiler or the process furnace (Fig.2);  $l = 2.5b$ ,  $h = b$ , range of  $b$  variation – up to 10 m, step of the size change – 2 m – for the industrial reheating furnace.

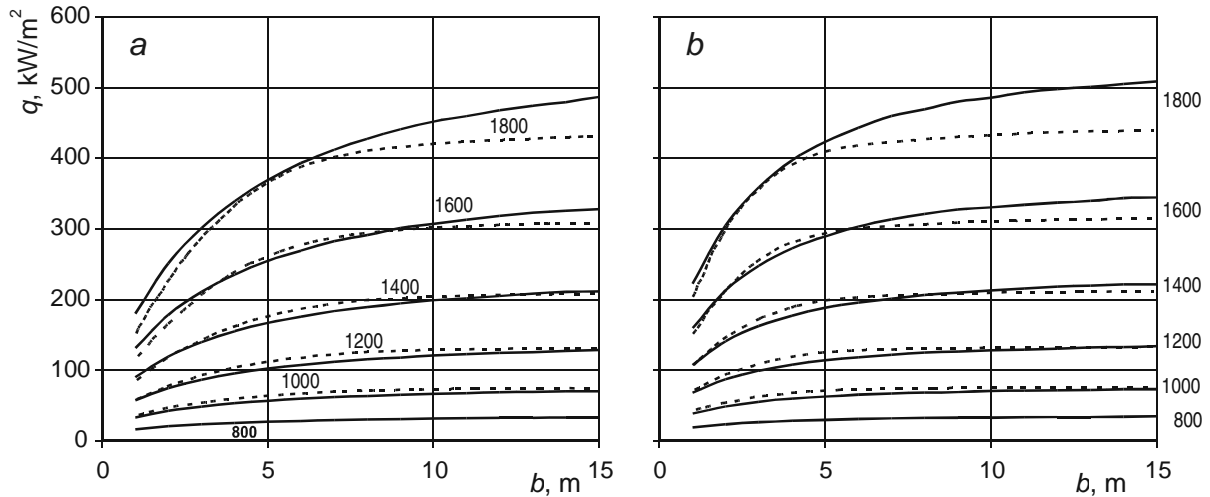


Fig. 2. Resulting radiative heat flux density  $q$  absorbed by the receiving surface within the boiler or process furnace in dependence on the combustion chamber width  $b$ .

Solid lines – our calculations by Monte-Carlo procedure, dotted lines – by H. Kremer [13, Fig.15]

Number of computation zones  $N_{cz} = 1$ .

Zone method computations parameter – furnace gas temperature  $T_g$ , °C, is indicated near the curves.

$O_2$  content in oxidant  $[O_2]$ , %: a– 30%; b– 50%

The following initial conditions are assumed by calculations: natural gas (methane) is considered as a fuel, air or air enriched with oxygen ( $O_2$  fraction in an oxidiser  $x_{O_2,0}$ ) - as an oxidizer, supplied for combustion by oxidant excess factor till 1.1 (the residual  $O_2$  contents in dry combustion products of 2 %). A range of a variation  $[O_2]$  in an oxidizer  $x_{O_2,0} = 0.21; 0.3; 0.5; 1.0$ , that corresponds to the sum of partial pressures of  $CO_2$  and of  $H_2O$  in dry combustion products  $p_{\Sigma} = p_{CO_2} + p_{H_2O}$ , resulted in Table 2.

Table 2. Partial pressure of the emitting components in combustion products of natural gas

$x_{O_2,0}$ , %	$\lambda$	$p_{\Sigma}$ , atm, for combustion products
20.95	1.094	0.262
30	1.061	0.372
50	1.031	0.585
100	1.01	0.993

H. Kremer has executed alternative furnace calculations by means of zone method approach with acceptance of the optical characteristics of the solid surfaces and of gaseous combustion products, being corresponded to grey radiation. Combustion products temperature  $T_g = 800 \dots 1800$  °C is settled as uniform value by furnace space. The step of variation of the last by computations was stated of 200 °C.

It was supposed, that in case of the boiler or the process furnace all walls are arranged with the tube heat receivers of the set temperature  $T_w = 400$  °C. In case of the industrial furnace heat receiver surface of the assigned temperature  $T_M = 400 \dots 1200$  °C occupies

the area  $2.5 \cdot b^2$  in the bottom base of the furnace presented as a parallelepiped. The step of 400 °C of variation of receiver temperature was assumed by computations. Other five side surfaces and the roof of the furnace are covered of refractory bricks. Specific heat loss through the walls (density of heat flux) was assumed of 5 kW/m<sup>2</sup>.

The MCP was realized by account of "history" of  $10^4$  beams being emitted by each of surface zones and of  $10^6$  beams let off by volume zones on each of  $n$  quasi-levels of radiation (model of the weighted sum of grey gases) by fulfilment of the boiler (or process furnace) calculations in case of the furnace space consideration similar to 1-zone system. Number of the beams have been taken into account makes:  $10^3$  – for each of surface zones,  $10^5$  – for each of volume zones - in case of division of the boiler (furnace) space into 54 volume zones.

Number of the beams taken into account by emission on each of  $n$  quasi-levels for any of volume and each of surface zones made  $80 \cdot 10^3$  while using of MCP by calculation of the industrial reheating furnace.

Comparison of the calculations results obtained by means of Monte-Carlo method with those computed by zone method for two objects – the boiler (the process furnace) (Fig.2) and the industrial reheating furnace – shows an increase in divergences of the resulting heat fluxes density  $q$  as the characteristic size of the unit or O<sub>2</sub> fraction in an oxidizer is rising. It is possible to assume, that the certain explanation of such character of a divergence of the results gives an account of the limited range of optical thickness of the gas volume, at which  $a_n$  and  $K_n$  factors were chosen by generalization of the data on emissivity factor  $\varepsilon_g$ .

Really, area of  $\varepsilon_g$  presentation (and  $A_g$  - the same) falls at parameter  $p_{CO_2} I_{ef} = 0.001 \dots 1.0 \text{ atm} \cdot m$  values according to [10]. Meanwhile the calculations were carried out for the furnaces of the characteristic sizes  $I_{ef} = 3.6 \frac{V}{\sum F} = 0.75 \dots 10.5 \text{ m}$ .

By considered conditions of combustion of natural gas with nitrogen-oxygen oxidizers, including an air ([O<sub>2</sub>]=20.95%) till pure [O<sub>2</sub>] (100%) at  $\lambda$  up to 1.1,  $p_{CO_2} I_{ef}$  under consideration varies within the range of values 0.065...3.255 atm·m. Thus the parameter  $p_{CO_2} I_{ef}$  in our computations exceeds the top limit of the parameter values being met to Taylor and Foster's recommendations [10] on use of the  $a_n$  and  $K_n$  values providing the real  $\varepsilon_g$  and  $A_g$  values. As a result the WSGG model doesn't fit to maximum sizes of the furnace under consideration.

**2.2.** E. Scholand has generalized the results of calculation of the tube cracking furnace (Fig.3) in the paper [14], executed by three methods: a – accordingly to the flux model, b – by Hottel's zone method and, c – by means of his own Scholand's simplified one-dimensional zone model. The last assumes division of the furnace under consideration by separate volume zones. These volume cells are submitted to computation by the perfect stirring reactor (PSR) model. All of the zones are calculated in combination by solution of the matrix formed by the system of the linear equations concerning effective fluxes from the surfaces enclosing the volume elements. Reciprocal heat and mass transfer between the vertical zones hasn't taken into account.

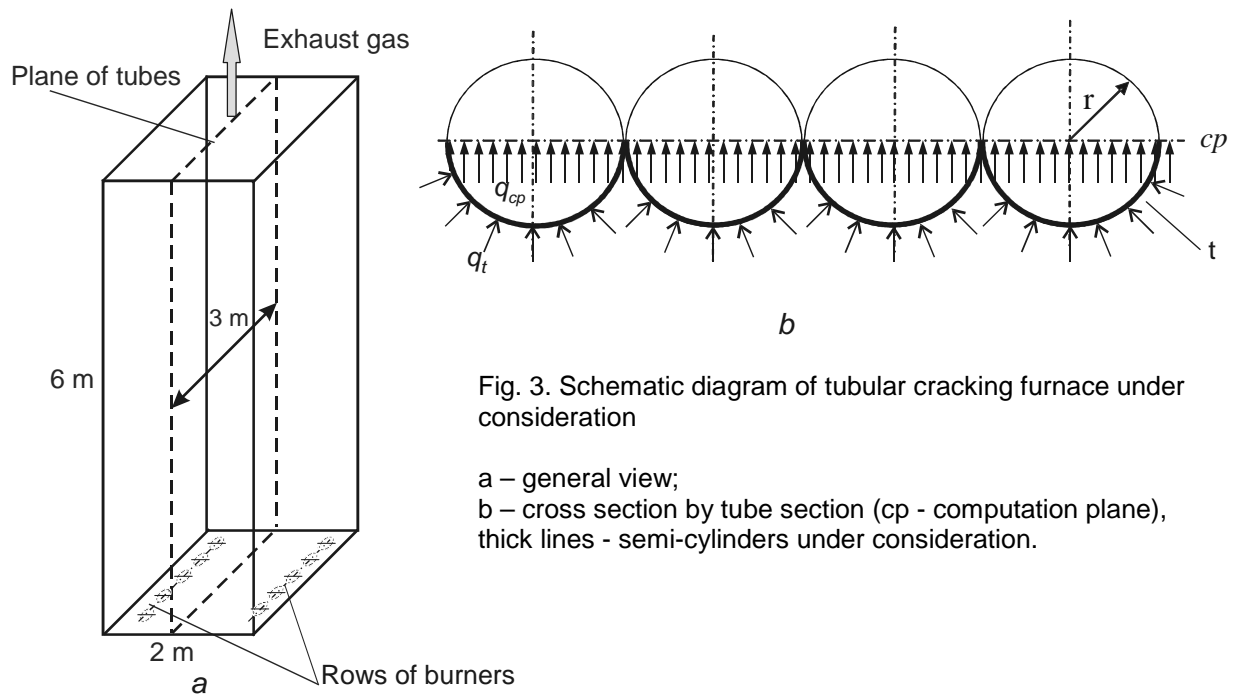


Fig. 3. Schematic diagram of tubular cracking furnace under consideration

a – general view;  
 b – cross section by tube section (cp - computation plane),  
 thick lines - semi-cylinders under consideration.

The initial data taken for calculation: fuel – the gas mixture of 70 % CH<sub>4</sub>, of 25 % H<sub>2</sub>, of 5 % N<sub>2</sub>, being burnt with an oxidizer – an air of excess air factor  $\lambda = 1.35$ . The tubes forming the reception surface are of an emissivity factor  $\varepsilon_w = 0.9$  and of temperature 1000 K. All other surfaces are considered as adiabatic ones. The temperature profile within the vertical planes of the furnace gas is assigned of six isothermic sections of the same height, while the horizontal cross-sections are completely met to the isothermal pattern of the combustions products. By the zone method calculations the furnace space was divided by height resulting in formation of six horizontal zones, of the size  $1 \times 1 \times 3 (m^3)$ . Another type of the zones formation supposes an additional division for calculations of each of the mentioned horizontal zones by three ones each of the last being of  $1 \times 1 \times 1 (m^3)$  in the sizes. Half of furnace space divided by an axial vertical plane  $F_{cp}$  was examined due to symmetry of the furnace space (Fig.3).

The data submitted in [21] testify to rather good conformity of the calculation results basing on zone method and on simplified one-dimensional method. Unlike mentioned methods the calculations by flux method have caused the strong deviation from their results.

Processing of the results submitted in [14] provides at the same time more than three times difference in the values obtained in separate sections of the furnace by calculations by flux method from those taken by zone method. The most reliable from the compared approaches is seemed apparently the H. Hottel's zone method.

Comparison of the mentioned results of calculation obtained in the paper [14] with our calculations, which have been carried out by Monte-Carlo method, is submitted in Fig.4. Division of the furnace space into the zones of two sizes mentioned above, as well as problem decision in two statements has been used: for grey and non-grey gas

radiation. In the first case the single value of medium absorption factor  $K = 0.2 \text{ m}^{-1}$  has been accepted for all volume of combustion products. In the second case the model of gas radiation as weighted sum of grey gases was used with acceptance of empirical factors  $a_n$  and  $k_n$  - accordingly data by P.B. Taylor, P.J. Foster [10].

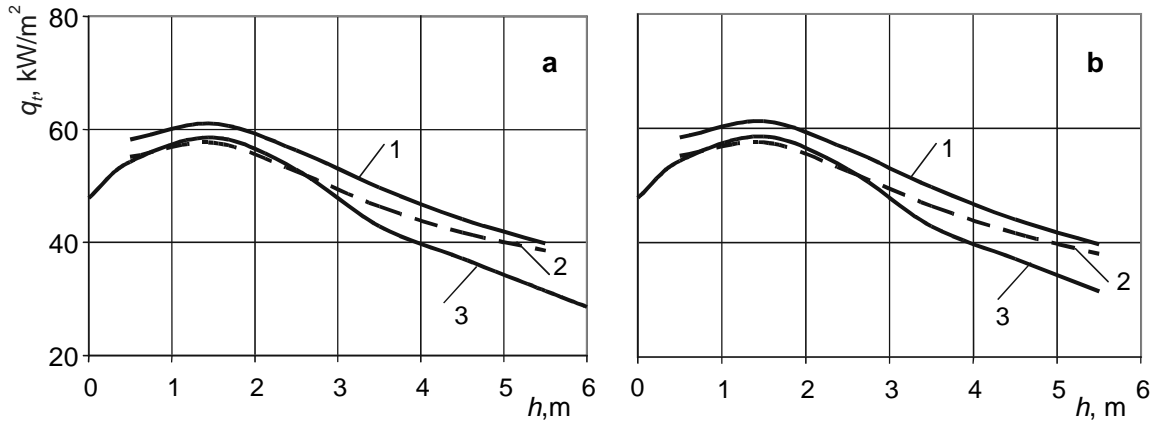


Fig. 4. Profile of resulting heat flux density  $q_t$  by furnace height  $h$  / Comparison of original predicted data with former calculations.

Number of computation zones/ zone sizes ( $\text{m}^3$ ):

a – 6 /  $1 \times 1 \times 3$ ; b– 18 /  $1 \times 1 \times 1$

1 – Monte-Carlo procedure, grey gas radiation,  $K=0.2 \text{ m}^{-1}$ ;

2 – Monte-Carlo procedure, non-grey gas radiation (WSGG)

$p_{\text{CO}_2} + p_{\text{H}_2\text{O}} = 0.22 \text{ atm}$ ,  $p_{\text{CO}_2} : p_{\text{H}_2\text{O}} = 2.36 : 1$ ;

3 – Zone method by H. Hottel, data summarized by E. Scholand and taken from [14].

The dynamics of power decrease by movement of 240000 beams by each zone at each of  $n$  radiation quasi-levels (WSGG model) was investigated by performance of Monte-Carlo procedure.

For recalculation of the computed values of heat fluxes directed to conditional reception surface  $F_{cp}$  in respect to true tube surface  $F_t$  the next equations were used:

$$Q_{cp} = q_{cp} F_{cp} = q_t F_t, \quad q_t = \frac{q_p}{0.5\pi} \quad (5)$$

where  $q_t$ ,  $q_p$  - heat flux density at the surface  $F_t$  of the tubes and within conditional plane  $F_{cp}$  - correspondingly.

It is possible to draw a conclusion from Fig.4 consideration, on good concurrence of the results of our calculations by Monte-Carlo procedure both for grey and non-grey gas emissivity models to those obtained by zone method, the results of our calculations for case of non-grey gas being in better response to the data resulted in the E. Scholand's paper for "grey" gas model of absorption coefficient  $K = 0.2 \text{ m}^{-1}$ . The maximum divergence of the results by Monte-Carlo model and by zone method use makes 14 % (min – 0 %).

Let's note in conclusion, that additional division of the isothermal zones does not result in change of the computation values of the heat fluxes (compare the data on  $q_t$  in Fig.4,a,b), that is connected apparently the advantages of developed generator of the random numbers by stochastic procedures.

## CONCLUSION

1. It's stated that Monte-Carlo method represents an operative computation means to predict both integral (and averaged within the furnace) and local thermal condition of the boiler furnace as well as of the separate facilities of the last with account of furnace design features and specifies of operational mode: by substitution the fuel and/or oxidant composition, of by input of supplementary components – recirculating combustion products, water vapour etc.

2. The Monte-Carlo method and procedures are developed for the furnace chamber, being divided into the computation zones. Selectivity of combustion products radiation within the furnace space is accounted by means of use of a weighted sum of emissivities of grey gases as a gas emissivity model, the weighting coefficients being dependent on temperature of emitting zone.

It was succeeded with this to investigate contribution into the resulting heat flux of each of heat transfer components: of the initial radiation of each zone (gas volume, surface) and of numerical reflections of the rays from the surfaces enclosing the furnace chamber or space.

3. Validation of the Monte-Carlo heat exchange computation method had been performed

- by use of the models of infinite flat layer – grounded on comparison with the results obtained by the method of multiple reflections. The method of presentation of the finite sizes flat layer surrounded by the side walls of mirror reflection to simulate of infinite flat layer by Monte-Carlo procedure calculation is developed.
- by computation of the boiler furnace and the oven chamber facilities – grounded on comparison with the zone method calculations.

The results of calculations of the boiler furnaces and of the industrial furnaces obtained by Monte-Carlo method were compared with the computations presented in some references, by the results of various existing methods: flux, zone 3D (three-dimensional) and simplified 1D (one-dimensional).

H. Hottel's zone method is the most perfect procedure of the existing methods of calculation of industrial furnaces of arbitrary geometry with account of gas radiation selectivity.

Monte-Carlo approach with respective procedures of its realization being proposed in frame of the paper provides qualitative and quantitative similarity to the results of 3D zone method computations but has some advantages from the standpoint of the method simplicity and of opportunity to determine the detailed heat pattern within the boiler furnace.

## REFERENCES

1. Zhuravlev Yu.A. Radiative heat exchange within firing engineering plants. – Krasnoyarsk: Krasnoyarsk Univ. Publ. House. – 1983. – 256 pp. (In Russian).
2. Arutyunov V.A., Bukhmirov V.V., Krupennikov S.A. Mathematical modeling of thermal operation of industrial furnaces // Sc. editor V.A. Arutyunov. – Moscow: “Metalurgiya” Publ. House. – 1983. – 256 pp. (In Russian).
3. Lisienko V.G., Volkov V.V., Malikov Ju.K. Fuel use improvement and heat exchange control within metallurgical furnaces. – Moscow: “Metalurgiya” Publ. House. – 1988. – 231pp. (In Russian).
4. Boineau P., Copin C., Aquile F. Heat transfer modelling using advanced zone model based on a CFD code // 6<sup>th</sup> European Conference on Industrial Furnaces and Boilers: Preprints. – Estoril – Portugal. 02-05 April 2002. vol.III: Modelling of Furnaces and Combustion Systems. – 10 pp.
5. Lybaert P. From CFD to zonal furnace models: an overview of head treatment furnace modeling // 6<sup>th</sup> European Conference on Industrial Furnaces and Boilers: Preprints – Estoril – Portugal. 02-05 April 2002. – 14 pp.
6. Soroka B.S. Intensification of thermal processes within fuel furnaces. – Kiev: “Naukova dumka” Publ. House. – 1993. – 416 pp. (In Russian).
7. B. Soroka, K. Pyanykh, V. Zgursky, M. Khinkis, H. Abbasi, J. Rabovitser. Mathematical modeling of low-emission combustion processes basing upon Monte-Carlo procedures // Preprints of 5<sup>th</sup> European conference on industrial furnaces and boilers (11-14 April 2000). – Espinho-Porto (Portugal) – Vol. II. – P. 12.
8. Soroka B. et al. Heat - and Mass Transfer Processes in Low-Emission by NO<sub>x</sub> Boiler Furnaces under Condition of Two-Stage Gasburning and Reaction Products Recirculation. // Proceedings of Heat and Mass Transfer IV Minsk Intern. Forum , 22 – 26 May, 2000, - V.10. –P.436-445. (in Russian).
9. Strohle J., Coelho P.J., Schnell U., Hein K. A non-grey radiation model for the simulation of coal-fired furnaces // 6<sup>th</sup> European Conference on Industrial Furnaces and Boilers: Preprints. – Estoril – Portugal. 02-05 April 2002. vol.III: Modelling of Furnaces and Combustion Systems. – 10 pp.
10. Taylor P.B., Foster P.J. The total emissivities of luminous and non-luminous flames// Int. Journ. Heat & Mass transfer. – 1974, 1974, vol.17. - №12 - P.1591-1605.
11. Steward F.R., Cannon P. The calculation of radiative heat flux in a cylindrical furnace using the Monte-Carlo method / Int. Journal of Heat & Mass Transfer. - 1971. - Vol.14, No.2. - P.245-261.
12. Lisienko V.G., Volkov V.V., Goncharov A.L. Mathematical modeling of heat exchange within furnaces and assemblies. Kiev: “Naukova dumka” Publ. House. – 1984. – 230 pp. (In Russian).
13. Kremer H. Moglichkeiten und Grenzen der Strahlungswarmeubertragung in industriellen Gasfeurungen // GasWarme Int. – 1994. – J.43, H.10. – S.482-495.
14. Scholand E. Modern procedures for the calculation of radiant heat transfer in direct-fired tubed furnaces // Int. Chemical Engineering, 1983. - Vol. 23, № 4. - P.600-610.